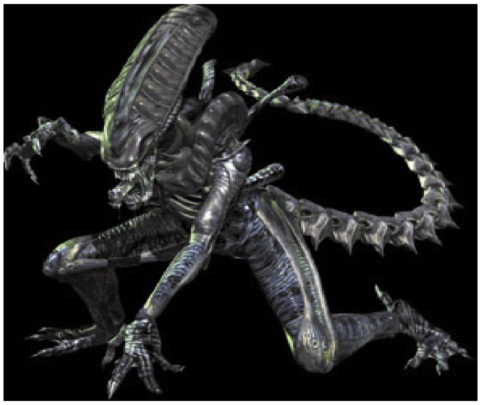
Monster Functions in Space Science I



Forget about the wimpy formulas you have played with before. Here is a reasonably complex formula that you will have to evaluate, and which will involve all the skills you have previously learned in algebra…and a mastery of scientific notation too!

Be careful, but don't be shy!

Keep track of your decimal points and exponents!!

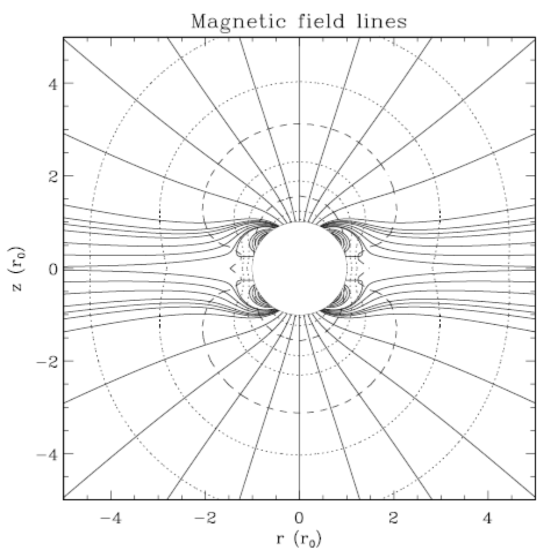
And, oh yes….Watch your back!!!

Abstract. We describe a simple analytic model for the magnetic field in the solar corona and interplanetary space which is appropriate to solar minimum conditions. The model combines an azimuthal current sheet in the equatorial plane with an axisymmetric multipole field representing the internal magnetic field of the Sun. The radial component of the field filling

From '*An Analytic Solar Magnetic Field Model*" by Banaszkiewicz, Axford and McKenzie (Astronomy and Astrophysics, vol. 337, p. 940-944.

$$\frac{B_\rho}{M} = \frac{3\rho z}{r^5} + \frac{15Q}{8}\frac{\rho z}{r^7}\frac{(4z^2-3\rho^2)}{r^2} + \frac{K}{a_1}\frac{\rho}{[(|z|+a_1)^2+\rho^2]^{3/2}}, \qquad (1)$$

$$\frac{B_z}{M} = \frac{2z^2-\rho^2}{r^5} + \frac{3Q}{8}\frac{(8z^4+3\rho^4-24\rho^2z^2)}{r^9} + \frac{K}{a_1}\frac{|z|+a_1}{[(|z|+a_1)^2+\rho^2]^{3/2}}, \qquad (2)$$



These formulas give the two components of the solar magnetic field, in units of Gauss, where **B** = $\mathbf{B}_\rho\ \rho + \mathbf{B}_z\ z$ where ρ and z are the unit vectors along these two directions.

<u>Problem 1:</u> Evaluate to the nearest tenth (B_ρ) and (B_z) for the following conditions appropriate to a distance from the sun equal to Earth's orbit using the following information:

$r^2 = \rho^2 + z^2$ K = 1.0

M = $6.03 \times 10^{+17}$ kilometers^3 Q = 1.5

$\alpha_1 = 1.07 \times 10^{+6}$ kilometers

where $z = -3.48 \times 10^7$ kilometers
$\rho = 1.46 \times 10^8$ kilometers.

<u>Problem 2:</u> Find the magnitude of the magnetic field strength using the values of the two computed components from Problem 1.

<u>Problem 3:</u> Explain what effect |z| has on plotting the magnetic field.

Answer Key:

$$\frac{B_\rho}{M} = \frac{3\rho z}{r^5} + \frac{15Q}{8}\frac{\rho z}{r^7}\frac{(4z^2 - 3\rho^2)}{r^2} + \frac{K}{a_1}\frac{\rho}{[(|z| + a_1)^2 + \rho^2]^{3/2}}, \quad (1)$$

$$\frac{B_z}{M} = \frac{2z^2 - \rho^2}{r^5} + \frac{3Q}{8}\frac{(8z^4 + 3\rho^4 - 24\rho^2 z^2)}{r^9} + \frac{K}{a_1}\frac{|z| + a_1}{[(|z| + a_1)^2 + \rho^2]^{3/2}}, \quad (2)$$

For $z = -3.48 \times 10^{7}$ kilometers $\rho = 1.46 \times 10^{8}$ kilometers.
Then $r = 1.5 \times 10^{8}$ kilometers………this equals the Earth-Sun orbital distance!

$$\text{Bp/M} = \frac{3\,(1.46 \times 10^{8})(-3.48 \times 10^{7})}{(1.5 \times 10^{8})^{5}} + \frac{15\,(1.5)}{8}\,\frac{(1.46 \times 10^{8})(-3.48 \times 10^{7})}{(1.5 \times 10^{8})^{7}}\,\frac{(4\,(-3.48 \times 10^{7})^{2} - 3\,(1.46 \times 10^{8})^{2})}{(1.5 \times 10^{8})^{2}}$$

$$+ \frac{1.0}{1.07 \times 10^{+6}}\,\frac{1.46 \times 10^{8}}{[\,(3.48 \times 10^{7} + 1.07 \times 10^{+6})^{2} + (1.46 \times 10^{8})^{2}\,]^{3/2}}$$

$$B_p = (6.03 \times 10^{+17})\,(-2.0 \times 10^{-25} + 2.3 \times 10^{-41} + 4.0 \times 10^{-23}) = 2.4 \times 10^{-5} \text{ Gauss}$$

$$\text{Bz/M} = \frac{2(-3.48 \times 10^{7})^{2} - (1.46 \times 10^{8})^{2}}{(1.5 \times 10^{8})^{5}} + \frac{3(1.5)}{8}\,\frac{[\,8(-3.48 \times 10^{7})^{4} + 3(1.5 \times 10^{8})^{4} - 24(1.46 \times 10^{8})^{2}(-3.48 \times 10^{7})^{2}]}{(1.5 \times 10^{8})^{9}}$$

$$+ \frac{1.0}{(1.07 \times 10^{+6})}\,\frac{(3.48 \times 10^{7} + 1.07 \times 10^{6})}{(\,(3.48 \times 10^{7} + 1.07 \times 10^{6})^{2} + (1.46 \times 10^{8})^{2}\,)^{3/2}}$$

$$B_z = (6.03 \times 10^{+17})\,(-2.5 \times 10^{-25} + 1.1 \times 10^{-41} + 9.8 \times 10^{-24}) = 5.8 \times 10^{-6} \text{ Gauss}$$

Problem 2: Use the Pythagorean Theorem to find B. $B = 2.5 \times 10^{-5}$ Gauss.

Problem 3: If you plot the value of B on the z-r plane, it will be symmetric along the z axis, reflected through a line at z=0. This is demonstrated in the figure on the front page of this problem.